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ADVANCES IN NIST DIELECTRIC MEASUREMENT CAPABILITY USING A MODE-FILTERED CYLINDRICAL CAVITY

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ABSTRACT

A 60-mm diameter cylindrical cavity resonator has been constructed for performing high-accuracy permittivity measurements on low-loss materials at microwave frequencies. The cavity's design and evaluation are described. Estimated errors in seven parameters result in approximately 0.2% uncertainty in permittivity and 6% uncertainty in loss tangent for a fused silica measurement.

INTRODUCTION

The National Institute of Standards and Technology (NIST) recognizes a national need for standardization of dielectric measurements. Engineers and scientists need to know the electromagnetic properties of many different materials in order to design microwave and millimeter wave devices. Presently many measurement techniques are in use. Some methods are well established, but many are relatively new, and have not been fully validated. Error analyses are usually very time consuming, and reference materials are very useful for validating a method being used. In response to industry's need, NIST has re-established an Electromagnetic Properties of Materials (EPM) program. NIST has identified several measurement techniques for development. The cavity resonator method was selected for early implementation because it is the most accurate method known for measuring low-loss dielectrics in the microwave and millimeter wave frequency range, and will enable NIST to resume the distribution of dielectric standard reference materials.

In the past seventeen years mode-filtered cylindrical resonators have been developed in the United Kingdom and the Federal Republic of Germany for measuring complex relative permittivity $\epsilon = \epsilon' - j\epsilon''$ [1,2]. The cylindrical walls of the cavity consist of helically wound wires which permit current to flow only in the circumferential direction. Non-TE_{01n} modes, if excited, will be highly attenuated. The mode filtering qualities of such a resonator permit identification of the principal TE_{01n} modes over a wide frequency range. In 1988 NIST designed and constructed such a helically wound cylindrical resonator. This paper presents that design and discusses the cavity performance and uncertainty analysis.

DESIGN

The cavity resonator is a 60-mm diameter cylinder with a fixed plate at one end and a moveable plate at the other. A cutaway assembly drawing of the device is shown in Figure 1. The inner wall of the cylindrical body consists of two 0.15 mm (0.006 in) diameter wires helically wound side by side. The fixed end plate has two small rectangular apertures through which energy is coupled from X-band waveguide into the cavity. The aperture sizes are designed to couple into the TE_{01n} modes and to yield approximately -30 dB resonance transmission (S_{21} or S_{12}) at 10 GHz. The fixed end plate is easily removable to allow for the study of different coupling schemes and frequency ranges. The micrometer-driven end plate allows cavity length to be varied from 408.5 mm to 433.5 mm. The moveable end plate is attached to three spring-loaded carbide rods which move on precision linear bearings. A motor-driven micrometer contacts the center of that end plate. The tuner assembly with the moveable end plate detaches from the cavity body and slides on support rods, allowing for insertion of a dielectric sample disk. The cavity body and tuner assembly are temperature controlled to maintain dimensional stability.

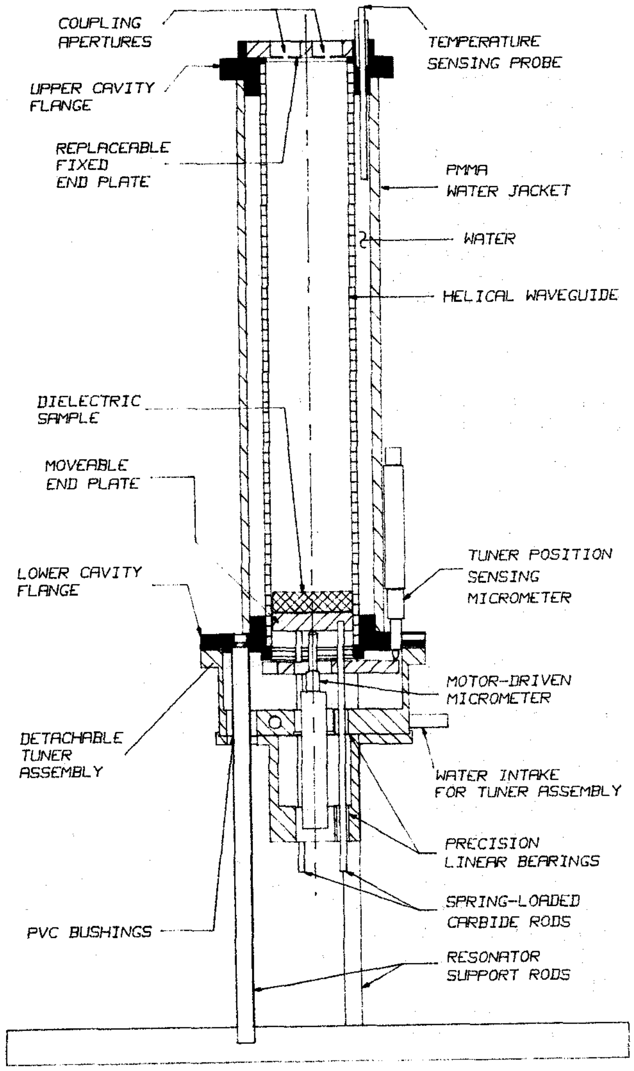

Figure 1: Cutaway View of NIST Resonator

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Permittivity (ϵ') and loss tangent ($\tan\delta = \epsilon''/\epsilon'$) calculations can be made by noting the change in a TE_{01n} mode when the resonator is empty versus when the resonator contains a sample. Either a fixed-length or a fixed-frequency technique may be used. The fixed-length technique uses the shift in resonance frequency and the change in resonance bandwidth when the sample is placed in the cavity to calculate ϵ' and $\tan\delta$. With the fixed-frequency technique, the cavity length is shortened so that the cavity resonates at the empty-cavity frequency. The change in cavity length and resonance bandwidth is used to calculate ϵ' and $\tan\delta$. To measure the change in length, the change in the moveable end plate position relative to the cavity body must be measured. This is accomplished using another micrometer attached to the bottom cavity flange.

THEORY

The theory for calculating permittivity using TE_{01n} modes is well documented [2,3]. The derivation for the real part, ϵ', of the complex permittivity neglects ohmic wall losses and coupling losses, and the sample is assumed to be low loss, linear with field strength, isotropic, and homogeneous. A resonance condition can be expressed by matching boundary conditions inside a cavity containing a sample. Multiple solutions to the resonance condition exist, and the correct ϵ' is found by choosing a starting value and applying the Newton-Raphson iterative method. According to [2], the resonance condition for the fixed-frequency technique is

$$\frac{\tan\beta_\epsilon b}{\beta_\epsilon} + \frac{\tan\beta_0(L - L_0 - b)}{\beta_0} = 0\ , \tag{1}$$

where β_ϵ and β_0 are the wave numbers in a cylindrical cavity containing a dielectric and air:

$$\beta_\epsilon^2 = \frac{\omega^2}{c_0^2}\mu_r\epsilon' - k^2\ , \tag{2}$$

$$\beta_0^2 = \frac{\omega^2}{c_0^2}\mu_{air}\epsilon_{air} - k^2\ , \text{ and} \tag{3}$$

$$k = \frac{j'_{02}}{r}\ . \tag{4}$$

The values for all parameters except ϵ' are known. The term $j'_{02} \cong 3.832$ is the first nonzero root of the derivative of the zero-order Bessel function $J'_0(j) = 0$; the speed of light in a vacuum is c_0; ω is the resonance frequency with sample in radians per second; the relative permeabilities, μ_r and μ_{air} are assumed equal to 1, the relative permittivity ϵ_{air} is calculated from the temperature, pressure and humidity in the laboratory [4]. The geometric definitions of the resonator are shown in Figure 2. The cavity radius is r, and b is the sample length. The change in cavity length required to re-tune the cavity to its original frequency after insertion of the dielectric sample is $L - L_0$. When the fixed-length technique is used, $L - L_0$ is replaced by the cavity length L in (1).

For the fixed-frequency technique, loss tangent is calculated according to [2]:

$$\tan\delta = \frac{(Ah_\epsilon - Dh_0)}{f_\epsilon}\ , \tag{5}$$

where f_ϵ is the resonance frequency with sample in hertzes, h_ϵ and h_0 are respectively the half-power resonance bandwidths for identical resonators that contain a lossy test specimen and a fictitious loss-free specimen of the same permittivity and dimensions. In (5),

$$A = \frac{2x\epsilon' + (B - \epsilon')\sin 2x + (2\beta_\epsilon L - 2x)P}{\epsilon'(2x - \sin 2x)}\ , \tag{6}$$

$$B = (\frac{\beta_\epsilon}{\beta_0})^2\ , \tag{7}$$

$$D = \frac{F + G}{E}\ , \tag{8}$$

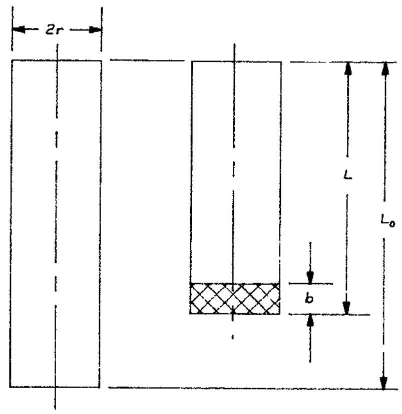

Figure 2: Geometric Definitions of Resonator

$$E = \epsilon'(q^2 + \frac{2a(1-q^2)}{L_0})\ , \tag{9}$$

$$F = q^2\frac{2x + (B-1)\sin 2x + (2\beta_\epsilon L - 2x)P}{2x - \sin 2x}\ , \tag{10}$$

$$G = 2\beta_\epsilon r\frac{(\epsilon' - q^2) + (1 - q^2)P}{2x - \sin 2x}\ , \tag{11}$$

$$P = \sin^2 x + B\cos^2 x\ , \tag{12}$$

$$x = \beta_\epsilon b = \chi\pi\ , \text{ and} \tag{13}$$

$$q = \frac{j'_{02}c_0}{2\pi r f_\epsilon}\ . \tag{14}$$

In (13), χ is the number of axial half-wavelengths in the sample. The derivation for loss tangent is based on assuming that the resonator has finite wall and coupling losses, and requires h_0 to be interpolated from the empty resonance spectrum. When the fixed-length technique is employed, L_0 is replaced by L in (9).

CAVITY EVALUATION

A number of experiments are being made to establish the resonator's performance and to help determine uncertainties in the measured parameters such as cavity length and diameter, resonance frequency and bandwidth, and sample length. The experiments show excellent measurement precision. At present, accuracy statements are preliminary, since further experimentation is needed to assess various corrections and to determine random and systematic uncertainties. This section presents the results of cavity evaluation experiments made thus far. Sources of errors are discussed, and preliminary uncertainties are used in the next section to calculate ϵ' and $\tan\delta$ uncertainties for measurements on fused silica.

Accurate knowledge of the cavity radius and length is critical to the calculation of ϵ' and $\tan\delta$. Cavity dimensions and their uncertainties are determined by applying a linear regression to the empty cavity TE_{01n} mode spectrum, where

$$f^2 = \frac{c_0^2}{4\epsilon_{air}}(\frac{j'^2_{02}}{\pi^2 r^2} + \frac{n^2}{L_0^2})\ . \tag{15}$$

To place a sample into the cavity, the tuner assembly must be opened and reattached to the cavity body. Table 1 shows that a length repeatability of approximately ±0.3 μm can be achieved at X-band frequencies. Table 1 also shows how frequency and length are related. The cavity's dimensions are temperature dependent, which requires close temperature control of the cavity body and the detachable tuner assembly. The water in the jacket surrounding the cavity body is monitored and

Mode Number	Frequency (GHz)	Frequency Tuning Rate (Hz/micron)	Frequency Repeatability (Hz)	Length Repeatability (microns)
16	8.227 840 605	8500	+/- 1914	+/- 0.23
23	10.017 308 950	14300	+/- 4160	+/- 0.29
27	11.147 429 220	17400	+/- 4610	+/- 0.27
31	12.329 239 280	21000	+/- 5320	+/- 0.26

Table 1: Length/Frequency Repeatability of Tuner Assembly

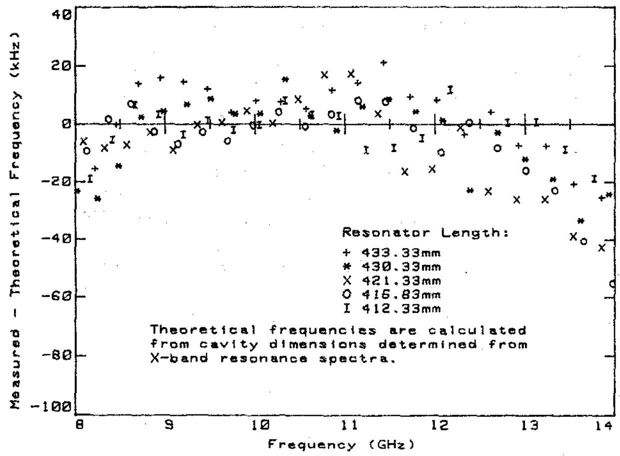

Figure 3: Frequency Shift at Various Resonator Lengths

controlled within 0.05°C. Water also circulates in passages in the tuner assembly. Experiments have shown that this temperature uncertainty translates into small uncertainties of ±0.24 nm in length and ±0.021 nm in diameter.

Although cavity length and diameter can be precisely determined by applying a linear regression to the empty-cavity mode spectrum, the calculations exhibit errors. Perturbational corrections to the resonant frequencies are needed to accurately calculate cavity dimensions. Figure 3 shows the difference in measured and theoretical frequency for five resonator lengths. The measured frequencies above 12 GHz are less than the theoretically calculated values. The effect of this frequency shift can be seen by using five adjacent modes to calculate cavity dimensions, using mode numbers 16–20, 17–21, ... , 33–37, as shown in Figure 4.

The causes of these frequency shifts are under investigation. As frequency increases, the electrical size of the coupling apertures in-

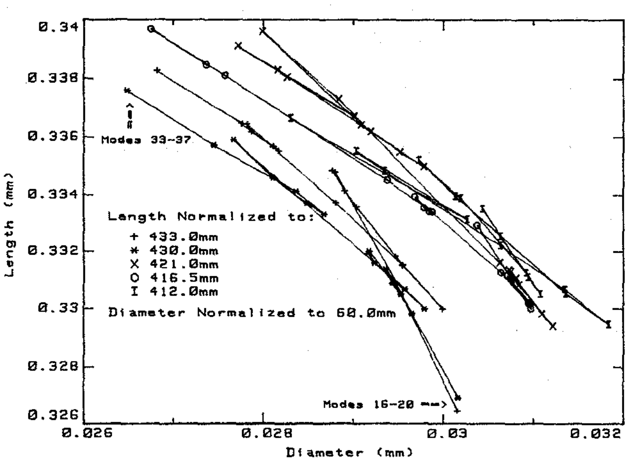

Figure 4: Calculated Dimensions Using 8–14 GHz Resonances

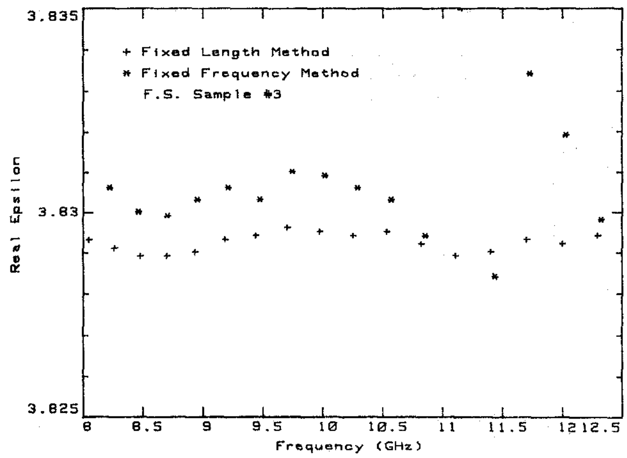

Figure 5: Permittivity Results of a Fused Silica Sample

creases, which increases coupling efficiency into and out of the cavity. The changing impedance of the apertures can re-tune the cavity. Also, mismatch at the coax-to-waveguide adapters causes resonance frequency shifts on the order of ±250 kHz at 12 GHz. This happens because the impedance at the apertures contains an additional reactive component caused by reflections from the adapter. At the time this paper was written, this frequency-shift problem was eliminated by inserting isolators into both the feed and detection waveguide systems. Other frequency shifts are caused by skin-depth losses on the end plates and nonideal cylindrical walls. These losses causes a downward resonance frequency shift on the order of half the resonance bandwidth [5,6]. A downward frequency shift makes the calculated cavity dimensions larger than its true size. These errors translate into a length uncertainty of approximately ±0.009 mm and a diameter uncertainty of approximately ±0.002 mm, which we will use in the next section.

In the derivation for $\tan\delta$, h_0 is assumed to be the resonance width of a cavity containing a fictitious loss-free specimen identical to that of a cavity containing a lossy specimen. The ficticious resonator system has identical dimensions and resonates at the same frequency as the actual lossy system. In practice, such a comparision can be approximated by interpolating empty-cavity resonance widths of adjacent modes [2]. If the fixed-frequency technique were used, the value for h_0 from the empty-cavity resonance spectrum would be greater than the needed value. Wall losses in the empty cavity are greater than in the ficticious cavity, because the empty cavity must lengthened to resonate at the same frequency, thereby exposing more lossy cavity wall to electromagnetic fields. This is especially true for highly perturbing samples where the change in length between the empty and loaded resonances becomes significant. A similar argument holds for the fixed-length technique. The empty-cavity resonance frequency is higher than the sample-loaded frequency, and cavity coupling losses become higher as frequency increases. With both techniques the value of h_0 determined from the empty-cavity resonance spectrum is greater than the necessary h_0, so calculated loss tangent values will be artificially low.

CHECK-STANDARD MEASUREMENTS

NIST has begun measurements on six fused silica specimens, some of which will become check standards. Four of the specimens are made of recently manufactured fused silica. The other two are made of previously characterized standard reference materials [7,8] and will serve as transfer standards for this technique. Each fused silica disk has different ratings for inhomogeneity, inclusions, OH ion and trace element concentration. The sample lengths across the fused silica disks are uniform to ± 5.0 μm, and the faces are polished to optical quality. The results thus far show excellent measurement precision and demonstrate slight, repeatable differences between samples.

Figure 5 presents measurements of ϵ' at X-band for one of the

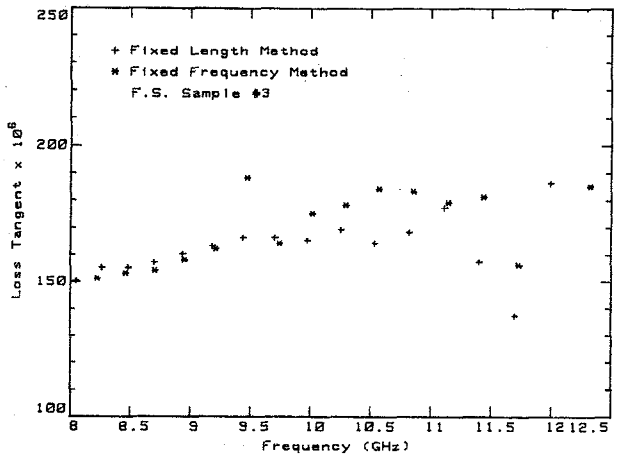

Figure 6: Loss Tangent Results of a Fused Silica Sample

Parameter	Value	Error	$\Delta\epsilon'$	$\Delta\tan\delta$ $\times 10^6$
r Cavity Radius (mm)	30.0148	0.001	0.0028	0.002
L Cavity Length (mm)	433.3404	0.009	0.0033	0.004
b Sample Length (mm)	14.4863	0.005	0.0004	0.308
c_0 Speed of Light (m/s)	299 713 300	351	0.0003	0.001
f_ϵ Frequency with Sample (GHz)	10.257 033	0.00005	0.0012	0.004
h_0 Resonance Bandwidth Empty Cavity (kHz)	131.85	1.2	XXX	3.8
h_ϵ Resonance Bandwidth with Sample (kHz)	186.07	1.9	XXX	5.9

Table 2: A Fixed-Length Fused Silica Measurement

fused silica samples. Results were calculated without corrections for frequency shifts, which will be developed later. The fixed-frequency compares well with the fixed-length technique, and the measurement precision for ϵ' is on the order of ± 0.002. Some results notably deviate from the mean value; these used TE_{01n} modes which were close to an interfering non-TE_{01n} mode. It has been shown that TE_{02n}, TE_{12n}, and TE_{13n} modes also exist, but at levels typically 30 dB lower than the dominant TE_{01n} modes. These deviations show that, while the non-TE_{01n} modes exhibit low unloaded Q-values, they still affect the measurement.

Figure 6 gives loss tangent results for the same fused silica sample. No corrections to the cavity dimensions or measured bandwidths have been applied. The results show very little scatter. Those results which significantly deviate from other values have been shown to be calculated from a resonance that is near a non-TE_{01n} mode. All of the fused silica specimens are very pure and have very low loss, but results seen thus far demonstrate sample-to-sample differences. Loss tangents for different samples range from $90\text{x}10^{-6}$ to $270\text{x}10^{-6}$. Most of these differences are probably due to manufacturing processes and slight sample impurities.

An error budget for one of the fixed-length measurements is given in Table 2. The uncertainties given for each parameter add estimates for error corrections and measurement uncertainties. In Table 2 the dependence of cavity dimensions on resonance frequencies has been ignored in order to give worst-case uncertainty. The uncertainties in resonance frequency and cavity dimensions can be separated to a large degree, but for this example are assumed to be independent. After correction for known errors, resonance frequency uncertainty due to S-parameter uncertainty, aperture impedances and skin-depth losses could be smaller than ± 3 kHz. Presently, the resonance frequencies can be measured with a precision higher than ± 500 Hz. The uncertainty in resonator dimensions can be separated from frequency uncertainty by correcting the frequency shifts before cavity dimension calculation and then considering only physical parameters such as tuner re-attachment repeatability and micrometer accuracy. After correction for frequency shifts and various losses the effective length accuracy could be as small as ± 2 μm and the effective diameter accuracy ± 0.6 μm.

CONCLUSION

The design, construction, and early evaluation of a new instrument for making dielectric measurements at about 10 GHz has been completed. The design is based on the use of a mode-filtered, circular waveguide for the walls of the resonant cavity. This results in high-Q TE_{01n} resonances (75 000 or greater) that are relatively free of interference from unwanted modes. The new instrument benefits from past NIST experience, as well as the advice and experience of workers in the United Kingdom and the Federal Republic of Germany. Complex permittivity measurements can now be made with an accuracy of approximately 0.5% in ϵ' and 10% in $\tan\delta$. With better knowledge of the cavity dimensions and error corrections, the accuracy of permittivity measurements might be improved by an order of magnitude.

When corrections for errors have been developed, this instrument will be used to characterize dielectric reference materials. These materials will be made available to other metrology laboratories in government and in industry. In addition, they will be used to evaluate other, less accurate dielectric measurement methods being implemented by the NIST Electromagnetic Properties of Materials program.

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REFERENCES

[1] R.J. Cook, "Microwave cavity methods," in *High Frequency Dielectric Measurement* (Conf. Proc., March 1972), J. Chamberlain and G.W. Chantry, Eds. Guildford, U.K., pp. 12–27: IPC Science and Technology Press, 1973.

[2] E. Ni and U. Stumper, "Permittivity measurements using a frequency-tuned microwave TE_{01} cavity resonator," *IEE Proceedings*, Vol. 132, Pt. H, No. 1, pp. 27–32, February 1985.

[3] F. Horner, T.A. Taylor, R. Dunsmuir, J. Lamb, and W. Jackson, "Resonance methods of dielectric measurement at centimetre wavelengths," *J. IEE*, Vol. 93, Pt. III, pp. 53–68, 1946.

[4] H.J. Liebe, "An updated model for millimeter wave propagation in moist air," *Radio Science*, Vol. 20, No. 5, pp. 1069–1089, September–October 1985.

[5] J.C. Slater, *Microwave Electronics*, D. Van Nostrand Company, Inc., New York (1950), pp. 75.

[6] J.D. Jackson, *Classical Electrodynamics, Second Edition*, J. Wiley and Sons, New York (1975), pp. 360.

[7] H.E. Bussey, J.E. Gray, E.C. Bamberger, E. Rushton, G. Russell, B.W. Petley, and D. Morris, "International comparision of dielectric measurements," *IEEE Trans. Instrum. Meas.*, Vol. IM-13, No. 4, pp. 305–311, December 1964.

[8] H.E. Bussey, D. Morris, and E.B. Zal'tsman, "International comparision of complex permittivity measurement at 9 GHz," *IEEE Trans. Instrum. Meas.*, Vol. IM-23, No. 3, pp. 235-239, September 1974.